Mathematical model and solution algorithm for packing problem of convex polyhedra

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Математична модель та алгоритм розв'язання задачі упаковки опуклих багатогранників

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***Abstract*—We study a packing problem of convex polyhedra into a container (rectangular, spherical or cylindrical) of minimal size. Continuous rotations and translations of polyhedra are allowed. We use radical free quasi-phi-functions and phi-functions to construct a mathematical model of the problem as NLP- problem. We also develop an efficient solution algorithm which allows us to reduce the problem dimension and computational cost. Several computational results are provided.**

Keywords—packing, convex polyhedra, continuous rotations, non-overlapping, mathematical model, nonlinear optimization

Optimal packing problem has a wide spectrum of applications in modern biology, mineralogy, medicine, materials science, nanotechnology, robotics, power engineering, mechanical engineering, shipbuilding, aircraft construction, civil engineering, etc. At present, the interest in finding effective solutions for packing problems is growing rapidly. This is due to a large and growing number of applications and an extreme complexity of methods used to handle many of them. These problems are NP-hard [1], and, as a result, solution methodologies generally employ heuristics [2-3]. We consider a problem of packing a collection of a given non-identical convex polyhedra into a container (rectangular, spherical or cylindrical) of minimal size (volume, radius or homothetic coefficient.

Our approach is based on mathematical modeling of relations between geometric objects and thus reducing the packing problem to a nonlinear programming problem. To this end we use the phi-function technique [4] for analytic description of non-overlapping and containment constraints taking into account their continuous rotations and translations. We apply the concept of phi-functions, extending their domains by including auxiliary variables. The new functions, called quasi-phi-functions [5], can be described by analytical formulas that are substantially simpler than those used for phi-functions, for some types of objects, in particular, for polyhedra.

One way to tackle the packing problem is use the existing phi-functions for rotating polyhedra. In the paper we propose alternative way to solve the problem which is based on quasi-phi-functions, is capable of finding a good local-optimal solution in reasonable computational time. The use of quasi-phi-functions, instead of phi-functions, allows us to simplify non-overlapping constraints, but there is a price to pay: now the optimization has to be performed over a larger set of parameters, including the extra variables used by our new functions, but this is a small price. We believe our quasi-phi-functions and our optimization algorithm described below are more flexible and efficient than other techniques.

We propose a mathematical model as a nonlinear programming problem by means of quasi-phi-functions and phi-functions and develop a solution algorithm, which involves a fast starting point algorithm and efficient local optimization procedures.

Our solution strategy involves the following steps:

1. Generate a number of starting points from the feasible region of our problem. We employ a new starting point algorithm.

2. Search for a local minimum of the objective function of our problem, starting from each point obtained at Step 1. We employ a special optimisation procedure – Local Optimization with Feasible Region Transformation.
3. Choose the best local minimum from those found at Step 2. This is our best solution of our problem.

We present our computational results for some new instances and several instances studied before. Figure 1 shows the local optimal placement of convex polyhedra in rectangular container found by our algorithm.

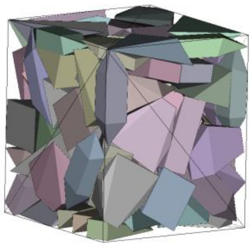

Fig.1. Local optimal placement of polyhedral rectangular container

Figure 2 shows the local optimal placement of convex polyhedra in cylindrical container found by our algorithm.

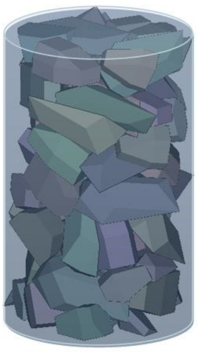

Fig.2. Local optimal placement of polyhedral cylindrical container

Figure 3 shows the local optimal placement of convex polyhedra in spherical container found by our algorithm.

Fig.3. Local optimal placement of polyhedral spherical container

CONCLUDING REMARKS

Using our radical free quasi-phi-functions and phi-functions we can develop exact nonlinear programming model for optimal packing of convex polytopes and applied our methodology to search for "good" local optimal solutions. The values of the objective function, as well as, the computational time reported in our paper for several examples is achieved presently, but we expect that it will be reduced in the future. The methodology may be extended for a case of non-convex polytopes.

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