Temperature Effect on a Non-Uniform Flexoelectric Wire with Electro-Thermo-Elastic Coupling

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Вплив Температури на Неоднорідний Флексоелектричний Дріт з Урахуванням Електротермопружного Взаємозв’язку

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***Abstract* — A one-dimensional mathematical model for temperature and force induced deformation of flexoelectric non-uniform microwires is derived from the Hamilton’s variation principle. To this end, a reformulated theory for isotropic dielectrics with flexoelectricity is employed. This non-classical theory incorporates the strain gradients, electric quadrupoles and flexoelectric effect. In order to solve the differential equations with variable coefficients, the differential quadrature method (DQM) is adopted. The effect of small-scale factor, flexoelectric material properties, microwire geometry and temperature changes on the coupled fields in stretched rods are studied.**

***Анотація* — Ґрунтуючись на варіаційному принципі Гамільтона, отримано одновимірну математичну модель флексоелектричного неоднорідного мікродроту під дією теплового та механічного навантаження. З цією метою використано модифіковану теорію для ізотропних діелектриків із флексоелектрикою. Ця некласична теорія включає градієнти деформацій, електричні квадруполі й флексоелектричний ефект. Для знаходження розв'язку диференціальних рівнянь зі змінними коефіцієнтами використано диференціальний квадратурний метод (DQM). Вивчено вплив мікроструктури, флексоелектричних властивостей матеріалу, профілю мікродроту й зміни температури на зв'язані поля в розтягнутих стрижнях.**

Keywords — non-classical theory; electro-thermo-mechanical coupling; flexoelectric effect; non-uniform wire.

Ключові слова — некласична теорія; електротермомеханічна взаємодія, флексоелектричний ефект; неоднорідний дріт.

I. Introduction

Over the past few decades, various studies have reported about the importance of coupling effects at the micro-/nanoscale. Although some results regarding the influence of temperature field on the elastic properties of solids at the nanoscale were obtained previously, there are no works on static electro-thermo-elastic problems in non-uniform isotropic rods with flexoelectricity. Inspired by this, the behavior of flexoelectric microwires with non-uniform cross section is investigated in this work.

II. Problem Formulation

Let us study deformation of a non-uniform isotropic flexoelectric microwire with length L and cross-sectional area $A(x)$, orientated along the x-axis. The wire is fixed at the end $x = 0$ with its side length a and subjected to a stretched load F_0 at its free end with side length b. Two edges of the wire are held at a constant but different temperature T_0 and $T_* = T_0 + \Delta T$. The side surfaces of the slender wire are heat-insulated and free of traction and surface charges. In order to study the influence of non-uniform profile on electroelastic behavior of microwire, we consider the special case: $A(x) = A_0\left(1 - cx/L\right)^2$, where A_0 is the cross-sectional area at $x = 0$, $c = 1 - b/a$, and $0 < c < 1$. Here, $A_0 = a^2/4$ for non-uniform wire with a square cross section and $A_0 = \pi a^2/4$ for wire with a circular cross section.

In order to capture the micro-stiffness and flexoelectric properties of the dielectrics, we employ the reformulated theory for isotropic dielectrics with flexoelectric effect [1]. Within this theory, the linear constitutive equations can be written as:

$$\sigma_{ij} = k\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon'_{ij} - f_1\delta_{ij}Q_{mm} - 2f_2Q_{ij} - \left(3k + 2\mu\right)\alpha_T\delta_{ij}\theta\,,$$

$$\tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk}\,,\quad p_i = 2\mu l_0^2\gamma_i + \left(f_1 + 2f_2/3\right)P_i\,,$$

$$m'_{ij} = 2\mu\left(l_2^2 + 9l_0^2/5\right)\chi'_{ij} + 2\mu\left(l_2^2 - 9l_0^2/5\right)\chi'_{ji} + 2f_2e_{ijk}P_k\,,$$

$$E_i = \alpha P_i + \left(f_1 + 2f_2/3\right)\gamma_i + 2f_2e_{ijk}\chi'_{jk}\,,$$

$$V_{ij} = \alpha\left(\delta_{ij}\beta_1^2Q_{mm} + \beta_2^2Q_{ij} + \beta_3^2Q_{ji}\right) - f_1\delta_{ij}\varepsilon_{mm} - 2f_2\varepsilon_{ij} - \zeta\delta_{ij}\theta\,,$$

where, σ_{ij} denotes the Cauchy stress, ε_{ij} is the strain, $\tau^{(1)}_{ijk}$, p_i, and m'_{ij} are the higher-order stresses, which are the work-conjugate to the deviatoric stretch gradient $\eta^{(1)}_{ijk}$, the dilatation gradient γ_i, and the deviatoric rotation gradient χ'_{ij}, respectively, E_i is the local electric field, V_{ij} represents the higher-order local electric field, $\theta = T - T_0$ is the temperature change, P_i denotes the electric polarization, $Q_{ij} = P_{i,j}$, e_{ipq} is the permutation symbol. Here, κ, μ, α_T and α are the classical material moduli, l_0, l_1, l_2 β_1, β_2, β_3, ζ, f_1 and f_2 are the higher-order material constants [1]. The components of the strain tensor and the higher-order metrics are:

$$\varepsilon_{ij} = \left(u_{i,j} + u_{j,i}\right)/2,\ \varepsilon'_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_{nn}/3,\ \gamma_i = \varepsilon_{mm,i},$$

$$\chi'_{ij} = e_{ipq}\varepsilon'_{jq,p},\ \eta^{(1)}_{ijk} = \left(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}\right)/3 - \Big[\delta_{ij}\left(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right)$$

$$+\delta_{jk}\left(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) + \delta_{ki}\left(\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right)\Big]/15,$$

where u_i is the component of mechanical displacement.

For considered boundary problem, a displacement and polarization vectors are: $\mathbf{u} = \left(u_x(x), 0, 0\right)$, $\mathbf{P} = \left(P_x(x), 0, 0\right)$. For stationary problems, the heat equation is not coupled with the rest equations governed the electro-elastic behavior of wire. Thus, the formulated problem can be solved consecutively. From the heat conduction equation and thermal boundary conditions $\theta(0) = 0$ and $\theta(L) = \Delta T$, we get the following temperature distribution for elastic wires with varying cross section: $\theta(x) = \Delta T\left(c^{-1} - 1\right)\left[\left(1 - cx/L\right)^{-1} - 1\right]$. Then, from the Hamilton's variational formulation, the governing linear system of differential equations can be obtained as:

$$\left[EAu_{x,x} - \frac{1}{2}\beta_T A\theta - fAP_{x,x} - 2\mu l^2\left(Au_{x,xx}\right)_{,x} - f\left(AP_x\right)_{,x}\right]_{,x} = 0,$$

$$A\left(\alpha P_x + \varphi_{,x} + fu_{x,xx}\right) - \alpha\beta^2\left(AP_{x,x}\right)_{,x} + f\left(Au_{x,x}\right)_{,x} + \frac{1}{2}\zeta\left(A\theta\right)_{,x} = 0,$$

$$\left[A\left(-\varepsilon_0\varphi_{,x} + P_x\right)\right]_{,x} = 0. \quad (1)$$

Here, φ is the electric potential, ε_0 = 8.854×10^{-12} C^2/(N m^2), $E = k + 4\mu/3$, $l^2 = \left(2l_1^2 + 9l_0^2\right)/5$, $\beta^2 = \beta_1^2 + \beta_2^2 + \beta_3^2$, $f = f_1 + 2f_2$. For analyzing of microwire behavior, we get a set of coupled differential equations (1) with variable coefficients.

III. Numerical Results and Conclusions

In order to solve the formulated boundary-value problem, the DQM is adopted [2]. In this method, the n-order derivative of a function $f(x)$ at a grid point x_i is considered as the weighted linear summation of the function values along that coordinate direction:

$$\frac{d^n f(x_i)}{dx^n} = \sum_{j=1}^{N} B^{(n)}_{ij} f(x_j),$$

where, N denotes the total number of discrete points distributed along the axial direction and $B^{(n)}_{ij}$ is the n-order weighting coefficients matrix. The grid points are adopted as the Chebyshev–Gauss–Lobatto points. Note that the used discretization technique leads to a set of ordinary equations from which the values $u_x(x_i)$, $P_x(x_i)$, and $\varphi(x_i)$ $\left(i = 1, 2, \dots N\right)$ can be found. The solution to the formulated boundary-value problem is founded for N = 100. Using a MATLAB Software, numerical calculations are carried out for polyvinylidene difluoride with the following material properties: E = 3.7 GPa, μ = 1.2 GPa, α = 1.38×10^{10} N m^2/C^2, α_T = 7.45×10^{-5} 1/K, f = 200 V, l = 1 μm and β = 2 μm. The tensile load is F_0 = 10 μN, the temperature change is set to be 2K.

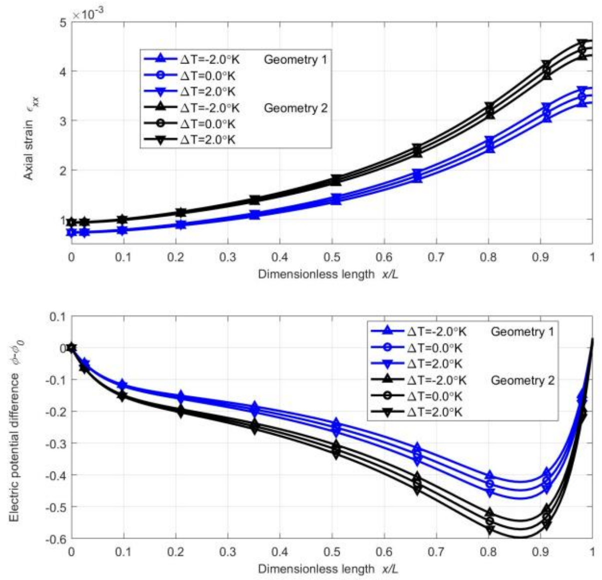

Fig. 1. The distributions of axial strain ε_{xx} and electric potential difference $\varphi - \varphi(0)$ in non-uniform wire (L = 12 μm, a = 2 μm, b = 0.8 μm) with square and circular cross sections (the blue and black lines, respectively) under complex effect of temperature and tensile loads.

The results of numerical calculations demonstrate the following. The disturbance of electric field in an isotropic wire appears due to the influence of both the varying rod profile and flexoelectric effect. The geometrical parameter c that denotes the taper ratio has a remarkable influence on the coupled electro-mechanical fields in the wire. The axial strain, displacement, polarization and absolute value of electric potential difference tend to increase with the increase of the taper ratio c. Axial strain and electric potential are sensitive to the sign and magnitude of temperature variation. With the increase of ΔT, the wire deformation and electric potential difference increases accordingly. The effect of rod profile more pronounced for the wires with circular cross section. The strain at left wire end tends to increase while the strain of right end of the rod tends to decrease when taking the effect of micro-stiffness into account.

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