Modeling of Filtration Processes in Media with Extended Horizontal Drains and Sources

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Моделювання Процесів Фільтрації в Середовищах з Довгими Горизонтальними Стоками та Джерелами

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***Abstract*—Filtration problems with drains and sources are considered in the porous media, where the drains and sources are modeled by delta-functions. Depending on coefficient magnitudes compared to the microscale parameter, three filtering modes are identified and described for the filtration problems. This classifies and simulates possible modes, for example, of modern oil production in problem wells, at least for the model inhomogeneous porous media under consideration. In all these cases, asymptotic approximations of solutions to such problems are obtained. Estimates of the accuracy of such asymptotic approximations are also presented. The estimates are necessary for computer simulation for the filtration problems with suitable accuracy. In turn, this makes it possible, instead of a direct (for example, numerical) solution to a rather complex problem for small microscales, to solve a simpler homogenized problem and receive a description of processes with guaranteed accuracy.**

***Анотація*—Проблеми фільтрації зі стоками та джерелами розглядаються в пористих середовищах, де стоки та джерела моделюються дельта-функціями. Залежно від величини коефіцієнта в порівнянні з мікромасштабним параметром ідентифіковано та описано три режими фільтрації для задач фільтрації. Це класифікує та моделює можливі режими, наприклад, сучасного видобутку нафти в проблемних свердловинах, принаймні для модельного неоднорідного пористого середовища, що розглядається. У всіх цих випадках отримані асимптотичні наближення розв'язків таких задач. Наведено також оцінки точності таких асимптотичних наближень. Ці оцінки необхідні для комп'ютерного моделювання задач фільтрації з належною точністю. Це, у свою чергу, дає змогу замість прямого (наприклад, чисельного) розв'язку досить складної задачі для малих мікромасштабів розв'язати більш просту гомогенізовану задачу й отримати моделей процесів із гарантованою точністю.**

***Keywords*—boundary value problems; homogenized problems; multiscale continuum; porous media**

***Ключові слова*—крайові задачі; гомогенізовані проблеми; багатомасштабний континуум; пористі середовища**

I. INTRODUCTION

Mathematical modeling of filtration processes in heterogeneous environments with extended horizontal drains and sources is important when planning optimal environmental management, developing methods for preventing technogenic pollution of groundwater and finding ways to purify such waters from pollution. So, recently it has become clear that extended horizontal wells are more efficient in oil and gas production. Thus, there is a need to study models of such processes and describe possible regimes for optimal purification of groundwater from pollution, oil production and gas production. Modeling such processes using direct methods of engineering observations is expensive and practically impossible, since to research filtration in real inhomogeneous porous media it is necessary to install a large number of sensors

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at different depths and large areas. Thus, mathematical modeling is the only way to investigate and optimize methods for rational extraction, purification and prevention of groundwater pollution. At the same time, through such extended horizontal wells it is possible to actually influence and optimize similar filtration and pumping processes that are used, for example, in gas and oil production. Some mathematical models of such influence in various modes will be discussed here.

In order to simulate filtration processes, it is natural to choose a suitable model of an inhomogeneous porous medium. Such media with a periodic structure are modeled most simply, since to describe such media it is enough to define only a periodicity cell and then periodically continue such a cell with suitable periods. This approach using periodic media makes it possible to quite accurately determine possible filtration modes and implement computer simulation of such processes. In turn, generalizations of this approach to random and stochastic continuous media are known, but the implementation of computer simulation in this case is problematic, since models of such media are based on infinite-dimensional implementations of random processes. It may be emphasized that homogeneous porous media are usually studied [1], the characteristics of which are obtained from considerations of some kind of averaging over volumes without any justification for the transitions.

II. Filtration in Media with Drains and Sources

Periodic media formed by a large number of cells, which may be inhomogeneous, will be considered. This approach defines a multiscale heterogeneous porous medium. Taking the multiscales into account, the modeling leads to the dependence of the coefficients of the equations on a small parameter characterizing the scales of the media and filtration ratio, which can determine different filtration regimes. This kind of research to problems with several parameters for periodic media is presented in [2], where further details can be found. This leads to homogenized (averaged) boundary value problems for equations with constant coefficients, the solutions of which are approximations of the solution to the problem for porous media (characterized by several parameters), as well as to estimates of the accuracy of such approximations for solutions to the considered problems.

Another approach to modeling filtration processes in periodic media is presented, for example, in [3], where a detailed bibliography may be found. With this approach, two-scale homogenized problems are established. However, the two-scale homogenized problems depend on two micro and macro variables and the type of such equations is not clear. Moreover, in this case, the accuracy of the two-scale approximation is unclear. Examples of direct numerical simulations for such filtration and heat transfer problems and comparisons with experiments are given in [4]. The optimization method for filtration problems with point sources proposed and studied in [5] and [6].

III. Filtration Modes in Periodic Continuous Media

Boundary value problems for second order differential elliptic equations with drains and sources will be considered. Such drains and sources in the filtration problems under consideration will be modeled by delta functions of suitable dimension. Here, results [7] on the regularity of solutions to these problems are significantly used. Additionally, when deriving homogenized models, the algorithm of asymptotic expansions for solutions presented in [2] is essentially used. In the resulting homogenized models, micro and macro variables are separated, which simplifies the understanding of possible filtering regimes in the problems under consideration. There can be three such filtering modes, depending on the asymptotic relationships of the coefficients of the equations of such problems. Namely, these are the modes of weak filtration and transfer, transfer and weak filtration, homogenized filtration and transfer. So, in all cases, instead of a direct solution of a complex problem for a small scale, one can solve homogenized problem and get a description of the model with guaranteed accuracy. When deriving models of these regimes, the filtration, transport and potentiality coefficients of the equations are taken into account in comparison with the scale of the inhomogeneous multiscale continuum.

Thus, the presented modes describe, respectively, weak transfer and filtration, homogenized transfer and weak filtration, homogenized filtration and transfer. This classifies and models the likely possible regimes, for example, of modern oil production in problematic wells, at least for the considered model of heterogeneous porous media. So, in the first case there is virtually no filtering or transfer. In the second, mass transfer occurs in a direction determined by homogenized coefficients. In the latter case, normal filtration and transfer are again defined by homogenized coefficients, as partially presented in [2]. This approach allows, instead of a direct (for example, numerical and computer) solution of a rather complex problem for small microscales, to solve a simpler homogenized problem and get process models with guaranteed accuracy. In turn, sometimes homogenized problems can be solved analytically in explicit form.

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